Calculus III, MiniTest 1 Review Answers

Dr. Graham-Squire, Fall 2013

1. Determine if the following points are collinear, and explain your answer:

$$P = (2, -1, 5) \qquad Q = (8, 3, 13) \qquad R = (-7, -7, -7).$$

Ans: Yes, because the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are scalar multiples.

2. An eagle with the head of Nido Qubein is pulling a rope attached to a statue of R Kelly across the quad. The eagle is pulling with a constant force of 100 pounds at an angle of 60° with the ground. If the eagle has to pull the statue 150 feet (so it can sit on a bench next to Gandhi, of course, where it belongs), find the amount of work done to get R Kelly to Gandhi.

Ans: 7500 foot-pounds.

3. When is $\mathbf{u} \cdot \mathbf{v} = 0$? When is $\mathbf{u} \times \mathbf{v} = \mathbf{0}$? Can you use a property or formula involving the cross and/or dot product to explain why that is the case?

Ans: $\mathbf{u} \cdot \mathbf{v} = 0$ when the two vectors are orthogonal (or when one of them is zero), and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ when the two vectors are parallel (or one of them is zero). You can convince yourself of this if you look at the formula for the angle between two vectors $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}$ and the magnitude of a cross product $||\mathbf{u} \times \mathbf{v}|| = ||\mathbf{u}||\,||\mathbf{v}||\sin(\theta)$.

4. Use vectors to find the area of the parallelogram that has sides given by the line segment connecting $(1, 2, -4)$ to $(2, 1, 3)$ and the line segment connecting $(1, 2, -4)$ to $(5, -1, 0)$.

Ans: $\sqrt{866}$

5. Sketch the plane given by the equation $x + 2y + 3z = 6$.

Ans: Do this on Sage or Grapher to check your work.

6. Find the distance between the parallel lines with parametric equations:

$$x = 3 + 2t \qquad y = t \qquad z = 4t - 3$$

and

$$x = 3 - 4t \qquad y = -1 - 2t \qquad z = 2 - 8t$$

Also explain how you know the two lines are parallel.

Ans: The two lines are parallel because if you look at the coefficients of the t to get your direction vectors, those vectors will be scalar multiples. The distance is $\sqrt{185/21}$.

7. Sketch the surface given by the equation $4x^2 - 9y^2 = -4z^2$.

Ans: This will be an elliptic cone with y as the rotational axis. You can either think of it as a quadric surface or a surface of revolution when you graph it.

8. (a) The point $(2, 2\pi/3, -2)$ is in cylindrical coordinates. Convert it to spherical coordinates.

Ans: $(2\sqrt{2}, 2\pi/3, 3\pi/4)$

(b) Find an equation in rectangular coordinates for the equation $z = r^2\cos^2\theta$ given in cylindrical coordinates. Sketch and/or describe the graph.

Ans: $z = x^2$. The graph will be a cylindrical surface with a parabola for a generating curve and rulings parallel to the y-axis.

9. (a) Describe the surface given (in cylindrical or spherical coordinates) by $\theta = \pi$. Explain your reasoning by explaining why your answer makes sense in cylindrical or spherical coordinates.

   Ans: The surface will be all points that have a θ value of π, with the other coordinates free. $\theta = \pi$ is the line in the direction of the negative x-axis, and if we allow any values for r and z (or ρ and ϕ, if you are thinking in spherical coordinates) then we will allow any points above and below the negative x-axis, as well as in the opposite direction (if r was less than zero, for example). So what we get is a plane that encompasses the x and z axes, which is the xz-plane.

   (b) Convert the equation to rectangular coordinates and compare your answer to what you got in part (a).

   Ans: Using the conversion equation $\tan(\theta) = \frac{y}{x}$ and substituting in $\theta = \pi$, we get $\tan(\pi) = \frac{y}{x}$. $\tan(\pi) = 0$, so we get $0 = \frac{y}{x}$. Multiplying x to the other side of the equation gives $y = 0$, which is the xz-plane in rectangular coordinates.